2015

TRIAL HSC
Examination

Name: ____________________

Teacher: ____________________

Year 12
Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen only
- Board approved calculators may be used
- A table of standard integrals is provided at the back of the paper
- In questions 11 – 16 show relevant mathematical reasoning and/or calculations

Section I ~ Pages 1-4

- 10 marks
- Attempt Questions 1-10
- Allow about 15 minutes for this section
- Use the Multiple Choice Answer Sheet

Section II ~ Pages 5 -13

- 90 marks
- Attempt Questions 11-16
- Allow about 2 hours 45 minutes for this section
- Answer each question in a separate writing book

Teachers:

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Examiner: Mr Mulray

Write your Name, your Board of Studies Student Number and your Teacher's Name on the front cover of each answer booklet

This paper MUST NOT be removed from the examination room.

Number of Students in Course: 85

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Section I

10 marks
Attempt questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

Question1

What is the value of $\dfrac{e^4}{7}$, correct to 3 significant figures?

(A) 7.78
(B) 7.79
(C) 7.790
(D) 7.80

Question 2

Which of the following is the solution of the quadratic equation $(1-2x)(3+x)=0$?

(A) $x=\frac{1}{2}$ and $x=3$
(B) $x=\frac{1}{2}$ and $x=-3$
(C) $x=-\frac{1}{2}$ and $x=-3$
(D) $x=-\frac{1}{2}$ and $x=3$

Question 3

What is the x coordinate of the point on the curve $y=e^{2x}$ where the tangent is parallel to the line $y=4x-1$?

(A) $x=\frac{1}{2}\ln 2$
(B) $x=\ln 2$
(C) $x=-\frac{1}{2}\ln 2$
(D) $x=2$

Question 4

A parabola has focus (– 4, 0) and directrix $x = 2$. What is the equation of the parabola?

(A) $y^2=-24(x+4)$
(B) $y^2=-12(x+1)$
(C) $y^2=24(x+4)$
(D) $y^2=12(x+1)$

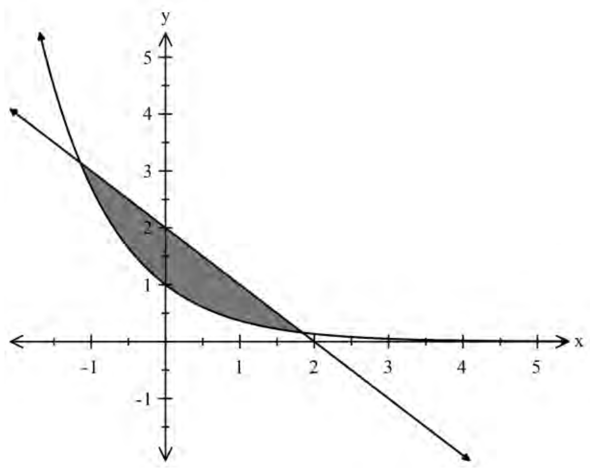
Question 5

A particle is moving along the x axis. The displacement of the particle after t seconds is given by $x = t^2 - 3t$ metres. Which statement describes the motion of the particle after 1 second?

(A) The particle is moving to the left with decreasing speed.
(B) The particle is moving to the right with decreasing speed.
(C) The particle is moving to the left with increasing speed.
(D) The particle is moving to the right with increasing speed.

Question 6

The diagram shows the region enclosed by $y = e^{-x}$ and $y = 2 - x$.



Which of the following pairs of inequalities describes the shaded region?

(A) $y \geq e^{-x}$ and $y \geq 2 - x$
(B) $y \geq e^{-x}$ and $y \leq 2 - x$
(C) $y \leq e^{-x}$ and $y \geq 2 - x$
(D) $y \leq e^{-x}$ and $y \leq 2 - x$

Question 7

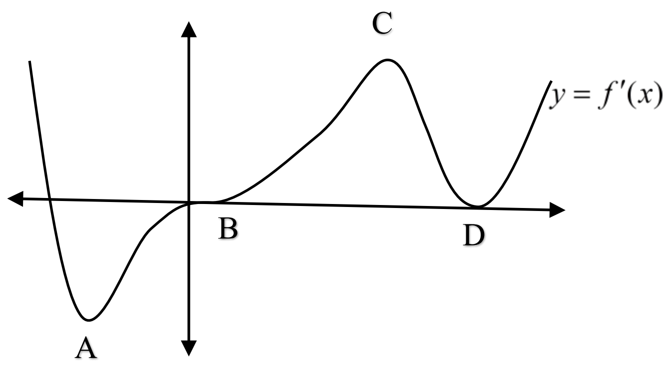
Consider the series $\sqrt{5} + 3\sqrt{5} + 5\sqrt{5} + ... = 225\sqrt{5}$

How many terms are in this series?

(A) 15
(B) 16
(C) 113
(D) 225

Question 8

The diagram shows a sketch of the gradient function $y = f'(x)$ passing through the points *A, B, C* and *D*.



Which point represents the horizontal point of inflexion of the curve $y = f(x)$?

(A) Point *A*
(B) Point *B*
(C) Point *C*
(D) Point *D*

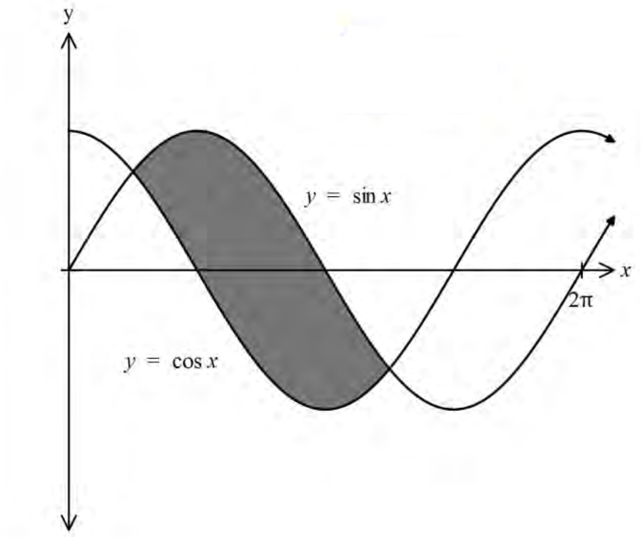
Question 9

What is the value of $f'(2)$ if $f(x) = \dfrac{1}{3x}$?

(A) $-\frac{1}{12}$
(B) $-\frac{1}{6}$
(C) $\frac{1}{3}$
(D) $-\frac{3}{4}$

Question 10

Which of the following definite integrals describes the shaded region?



(A) $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (\sin x - \cos x)dx$

(B) $\int_{\frac{\pi}{3}}^{\frac{4\pi}{3}} (\sin x - \cos x)dx$

(C) $\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)dx$

(D) $2\int_{\frac{\pi}{4}}^{\pi} (\sin x - \cos x)dx$

End of Section I

Section II

90 marks
Attempt questions 11 – 16
Allow about 2 hours 45 minutes for this section
Answer each question in a separate writing booklet. Extra writing booklets are available.
All necessary working should be shown in every question.

Question 11 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Factorise $6x^2 - 11x - 2$. **2**

(b) If $a + \sqrt{b} = (3 + \sqrt{2})^2$, find the values of a and b. **2**

(c) Solve $|1 - 3x| > 1$ **2**

(d) Find a primitive function of $2 - \sqrt{x}$. **2**

(e) Differentiate $\frac{1}{\cos x}$ **2**

(f) The gradient function of a curve $y = f(x)$ is given by $f'(x) = e^{2x}$. The curve passes through the point $\left(0, -\frac{1}{2}\right)$. Find the equation of the curve. **2**

(g) Evaluate $\int_1^e 1 + \frac{1}{x}\, dx$. **3**

End of Question 11

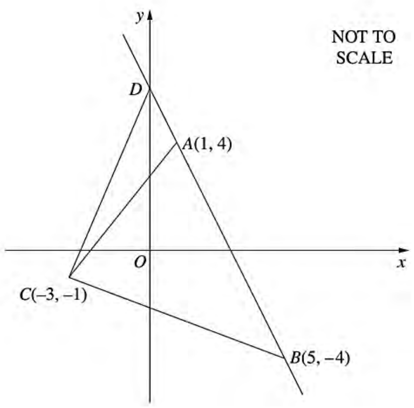
Question 12 (15 marks) Use a SEPARATE Writing Booklet. **Marks**

(a) The quadratic equation $4x^2 - 3x - 2 = 0$ has the roots α and β. Find:

(i) $\alpha + \beta$ **1**

(ii) $\alpha^3\beta^2 + \alpha^2\beta^3$ **2**

(b) *A*, *B* and *C* are the points $(1, 4)$, $(5, -4)$ and $(-3, -1)$ respectively, as shown in the diagram. The line *AB* meets the *y* axis at *D*.



(i) Show that the equation of the line *AB* is $2x + y - 6 = 0$ **2**

(ii) Find the coordinates of the point *D*. **1**

(iii) Find the perpendicular distance of the point *C* from the line *AB*. **2**

(iv) Hence, or otherwise, find the area of the triangle *ADC*. **2**

Question 12 continues on next page

(c) The amount of precious metal mined by a small company in each of the first three months of operation were 4000grams, 3920 grams and 3840 grams respectively. The pattern continues throughout the operation of the company. The mine runs out of precious metal after 50 months.

(i) How many grams were mined in the 12^{th} month? **1**

(ii) How many grams were mined over the first year? **1**

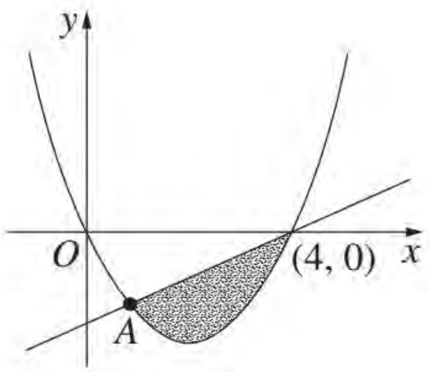
(iii) The mining company placed 25% of the precious metal mined each month into storage for future investment. The company sells the remaining 75% to an overseas company each month. That is 3000 grams, 2940grams and 2880g was sold in the first three months respectively. How many months does the company need to mine to sell a total of 73.2 kg to the overseas company? **3**

End of Question 12

Question 13 (15 marks) Use a SEPARATE Writing Booklet.

(a) Use Simpson's rule with three function values to find an approximation for $\int_2^6 \frac{x}{\ln x}\,dx$, give your answer correct to 1 decimal place. **3**

(b) The graph of $y = x - 4$ and $y = x^2 - 4x$ intersect at the point (4, 0) and A, as shown in the diagram.



(i) Find the x coordinate of A. **1**

(ii) Find the area of the shaded region bounded by $y = x - 4$ and $y = x^2 - 4x$. **3**

(c) Tim and Peter play against each other in the third round of a tennis tournament. In this tournament a match can last three sets, the first player to win two sets wins the match. The probability that Tim wins any set is 70% and 30% for Peter. Find the probability that:

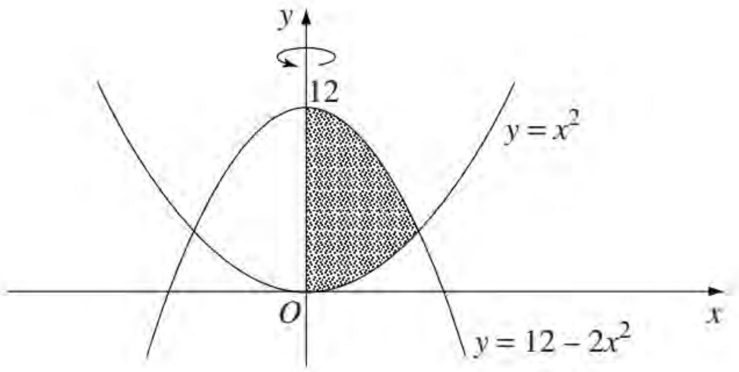
(i) The match will last 2 sets only. **2**

(ii) Tim wins the match. **2**

(iii) Peter wins the match. **1**

Question 13 continues on next page

(d) The graph of the curves $y = x^2$ and $y = 12 - 2x^2$ intersect at the points (– 2, 4) and (2, 4).



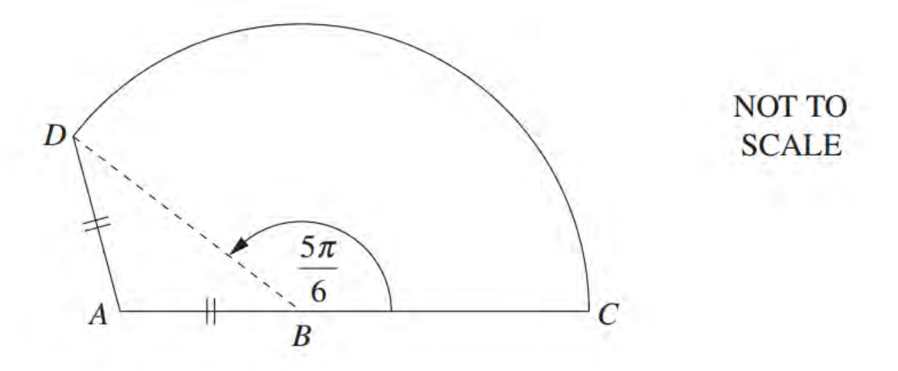
The shaded region between the curves and the *y* axis is rotated about the *y* axis. By the splitting the shaded region into two parts, or otherwise, find the exact volume of the solid formed. **3**

End of Question 13

Question 14 (15 marks) Use a SEPARATE Writing Booklet. **Marks**

(a) In the diagram, ABCD represents a garden. The sector BCD has centre B and $\angle DBC = \frac{5\pi}{6}$.
The points A, B and C lie on a straight line and AB = AD = 3 metres.

Copy or trace the diagram into your writing booklet.



(i) Show that $\angle DAB = \frac{2\pi}{3}$. **1**

(ii) Find the length of BD. **2**

(iii) Find the area of the garden ABCD. **2**

(b) Find the coordinates of the stationary point on the graph $y = \frac{\ln x}{x}$, $x > 0$ and determine its nature. **3**

(c) A particle moves in a straight line with acceleration after t seconds given by $a = 4\sin 2t$ m/s^2. Initially the particle is 1 metre to the left of the origin and travelling with a velocity of 2 m/s.

(i) Show that the velocity of the particle is given by $v = 4 - 2\cos 2t$. **2**

(ii) Show that the particle never comes to rest. **1**

(iii) Sketch the graph of velocity, v, as a function of time, t, for $0 \le t \le \pi$. **2**

(iv) Find the distance travelled by the particle in the first 4 seconds. Write your answer to the nearest metre. **2**

End of Question 14

Question 15 (15 marks) Use a SEPARATE Writing Booklet. **Marks**

(a) In the diagram, BE || CD and BE bisects $\angle ABD$. Copy or trace the diagram into your writing booklet.

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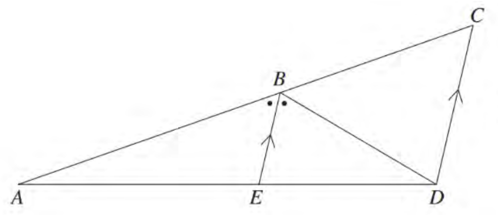


NOT TO SCALE

(i) Explain why $\angle EBD = \angle BDC$. **1**

(ii) Prove that ΔBCD is isosceles. **2**

(iii) Hence show that AE:ED = AB:BD. **2**

(b) A rare species of bird lives only in a remote island. A mathematical model predicts that the bird population, P, is given by $P = 150 + 300e^{-0.05t}$ where *t* is the number of years after observation began.

(i) According to the model, what will be the rate of change in the bird population ten years after observation began? **2**

(ii) What does the model predict will be the limiting value of the bird population? **1**

(iii) The species will become eligible for inclusion on the endangered species list when the population falls below 200. When does the model predict this will occur? **2**

(c) A property investor requires a loan of \$P from the bank to finance a purchase, with interest charged at an introductory rate of 6% p.a. for the first 3 months. Initially the loan is to be repaid in equal monthly repayments of \$4000 over 3 years and interest is charged monthly before each repayment.
Let $\$A_n$ be the amount owing after the *n*th repayment.

(i) Write down an expression for the amount owing after one month, $\$A_1$. **1**

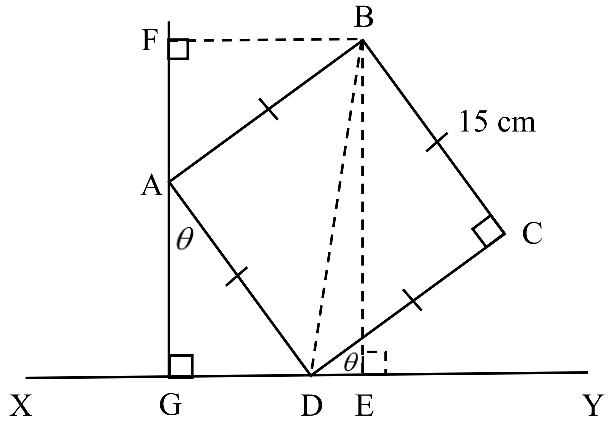
(ii) Show that $\$A_3 = P(1.005)^3 - 4000(1 + 1.005 + 1.005^2)$. **2**

(iii) At the end of three months the interest rate rises to 9% p.a. and the loan is to be repaid in total in equal monthly repayments of \$4800 for the next 2.75 years. Find the value of P. **2**

End of Question 15

Question 16 (15 marks) Use a SEPARATE Writing Booklet. **Marks**

(a) The diagram shows a square ABCD with sides 15cm leaning against a wall, FG, at an angle θ to the vertical and to the ground XY.



(i) Show that $BD = 15\sqrt{2}$ **1**

(ii) Hence show that $BE = 15\sqrt{2}\sin\left(\dfrac{\pi}{4} + \theta\right)$. **2**

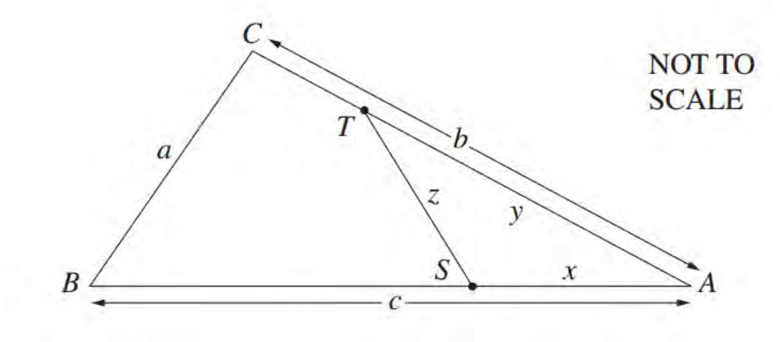
(iii) Find an expression for the length of FA and GA in terms of θ. **2**

(iv) Hence show that $\sin\theta + \cos\theta = \sqrt{2}\sin\left(\dfrac{\pi}{4} + \theta\right)$. **1**

Question 16 continues on next page

(b) The diagram shows a triangular piece of land ABC with dimensions AB = c metres, AC = b metres and BC = a metres where $a \le b \le c$.

The owner of the land wants to build a straight fence to divide the land into two pieces of equal area. Let S and T be the points on AB and AC respectively so that ST divides the land into two equal areas. Let AS = x metres, AT = y metres and ST = z metres.



(i) Show that $xy = \frac{1}{2}bc$. **1**

(ii) Use the cosine rule in triangle AST to show that $z^2 = x^2 + \frac{b^2c^2}{4x^2} - bc\cos A$. **2**

(iii) Show that the value of z^2 in the equation in part (ii) is a minimum when $x = \sqrt{\frac{bc}{2}}$. **3**

(iv) Show that the minimum length of the fence is $\sqrt{\frac{(P-2b)(P-2c)}{2}}$ metres, where $P = a+b+c$. (You may assume that the value of x given in part (iii) is feasible). **3**

End of paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x\ , \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

Note $\ln x = \log_e x, \quad x > 0$

SOLUTIONS 2UNIT 2015

Q1 D
Q2 B
Q3 A
Q4 B
Q5 A
Q6 B
Q7 A
Q8 D
Q9 A
Q10 C

DETAILED SOLUTIONS ON LAST PAGE. p 8

Question 11

(a) $6x^2 - 11x - 2 = (6x+1)(x-2)$ (2)

(b) $a + \sqrt{b} = (3+\sqrt{2})^2$

$= 9 + 6\sqrt{2} + 2$

$= 11 + 6\sqrt{2}$ (1)

$= 11 + \sqrt{72}$

$\therefore a = 11, \ b = 72$ (1)

(c) $|1-3x| > 1$

$1 - 3x < -1$ or $1 - 3x > 1$

$-3x < -2$ $\qquad$ $-3x > 0$

$x > 2/3$ (1) $\qquad$ $x < 0$ (1)

(d) primitive of $\sqrt{x} + \frac{1}{x}$

is $\frac{2}{3}x^{3/2} + \ln x$ (1) (1)

(e) $\frac{d}{dx}\frac{1}{\cos x} = \frac{d}{dx}(\cos x)^{-1}$

(1) $= -(\cos x)^{-2}\cdot(-\sin x)$

$= \sin x(\cos x)^{-2}$

(1) $= \dfrac{\sin x}{\cos^2 x}$

(f) $f'(x) = e^{2x}$

$f(x) = \dfrac{e^{2x}}{2} + c$ (1)

when $x = 0$, $y = -\frac{1}{2}$

$\therefore -\frac{1}{2} = \frac{1}{2} + c$

$\therefore c = -1$

$\therefore f(x) = \dfrac{e^{2x}}{2} - 1$ (1)

(g) $\displaystyle\int_1^e \left(1 + \frac{1}{x}\right) dx = \Big[x + \ln x\Big]_1^e$ (1)

$= (e + \ln e) - (1 + \ln 1)$ (1)

$= e + 1 - 1$

$= e$ (1)

Question 12.

(a) $4x^2 - 3x - 2 = 0$

(i) $\alpha + \beta = 3/4$ ①

(ii) $\alpha^3\beta^2 + \alpha^2\beta^3 = \alpha^2\beta^2(\alpha+\beta)$

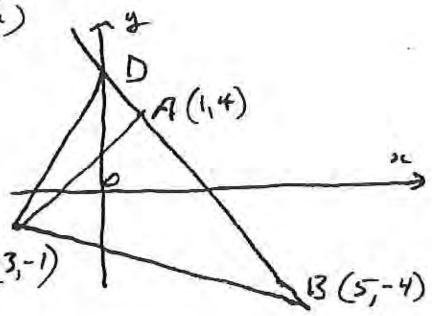
$= (\alpha\beta)^2(\alpha+\beta)$ ①

$= (-\frac{1}{2})^2(3/4)$

$= \frac{1}{4} \times \frac{3}{4}$

$= 3/16$ ①

(b)



(i) Gradient $AB = -2$ ①

Equ. line AB, $y - 4 = -2(x-1)$

$y - 4 = -2x + 2$ ①

$\therefore 2x + y - 6 = 0$

(ii) when $x = 0$, $2\times0 + y - 6 = 0$

$\therefore y = 6$

$\therefore D(0,6)$ ①

(iii) $2x + y - 6 = 0$, $C(-3,-1)$

$\perp d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2+B^2}}$

$= \frac{|2(-3) + 1(-1) - 6|}{\sqrt{2^2+1^2}}$ ①

$= \frac{|-13|}{\sqrt{5}}$

$= \frac{13}{\sqrt{5}}$ ①

(iv) $AD = \sqrt{(0-1)^2 + (6-4)^2}$

$= \sqrt{(-1)^2 + 2^2}$

$= \sqrt{5}$ ①

Area $\triangle ADC = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times \sqrt{5} \times \frac{13}{\sqrt{5}}$

$= 6\frac{1}{2}$ units^2. ①

(c) 4000, 3920, 3840, ...

AP, $a = 4000$, $d = -80$

(i) $T_{12} = a + 11d$

$= 4000 + 11 \times (-80)$

$= 3120$

3120g mined in the 12^{th} month ①

(ii) $S_{12} = \frac{n}{2}(a + l)$

$= 6(4000 + 3120)$

$= 42\,720$

42720g mined over the first year ①

(iii) $75\% \times 4000 = 3000$

$75\% \times 3920 = 2940$

$75\% \times 3840 = 2880$,

Sold each month, 3000, 2940, 2880, ...

$\therefore 73200 = \frac{n}{2}[2\times3000 + (n-1)(-60)]$ ①

$73200 = \frac{n}{2}[6000 - 60n + 60]$

$= n[3000 - 30n + 30]$

$= 3030n - 30n^2$

$\therefore 30n^2 - 3030n + 73200 = 0$ ①

$n^2 - 101n + 2440 = 0$

$(n - 40)(n - 61) = 0$

$n = 40$, $n \neq 61$ as $n \leq 50$,

Company can mine for 40 months. ①

Question 13.

(a) $f(x) = \dfrac{x}{\log_e x}$

x	2	4	6
$f(x)$	$\frac{2}{\log 2}$	$\frac{4}{\log 4}$	$\frac{6}{\log 6}$
	y_0	y_1	y_2

(1)

$$\int_2^6 \frac{x}{\log x}\,dx \doteq \frac{h}{3}\left[y_0 + y_2 + 4y_1\right]$$

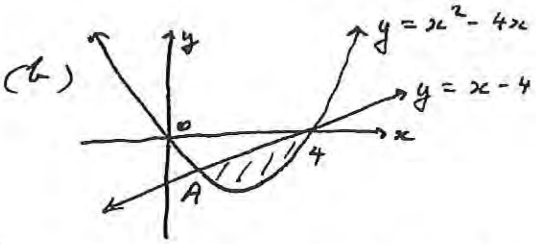
$$\doteq \frac{2}{3}\left[\frac{2}{\log 2} + \frac{6}{\log 6} + 4\left(\frac{4}{\log 4}\right)\right] \quad (1)$$

$$\doteq 11.8504\ldots$$

$= 11.9$ to one decimal place

(1)

(b)



(i) Solving simultaneously:

$y = x - 4$, $y = x^2 - 4x$

$x^2 - 4x = x - 4$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$\therefore$ x-coordinate of A is 1 (1)

(ii) Shaded area,

$$= \int_1^4 \left[(x-4) - (x^2 - 4x)\right] dx \quad (1)$$

$$= \int_1^4 (x - 4 - x^2 + 4x)\,dx$$

$$= \int_1^4 (5x - x^2 - 4)\,dx$$

$$= \left[\frac{5x^2}{2} - \frac{x^3}{3} - 4x\right]_1^4 \quad (1)$$

$$= \left[40 - \frac{64}{3} - 16\right] - \left[\frac{5}{2} - \frac{1}{3} - 4\right]$$

$= 4\frac{1}{2}$ units^2 (1)

c) (i) $P(T) = \frac{7}{10}$, $P(P) = \frac{3}{10}$

$$P(\text{2 sets only}) = P(TT) + P(PP) \quad (1)$$

$$= \frac{7}{10} \times \frac{7}{10} + \frac{3}{10} \times \frac{3}{10}$$

$$= \frac{29}{50} \quad (1)$$

(ii) $P(\text{Tim wins the match})$

$$= P(TT) + P(TPT) + P(PTT) \quad (1)$$

$$= \frac{7}{10} \times \frac{7}{10} + \frac{7}{10} \times \frac{3}{10} \times \frac{7}{10} + \frac{3}{10} \times \frac{7}{10} \times \frac{7}{10}$$

$$= \frac{784}{1000} \quad (1)$$

$$= \frac{98}{125}$$

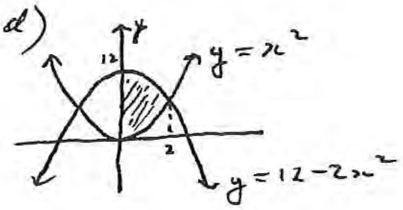
(iii) $P(\text{Peter wins the match})$

$$= 1 - P(\text{Tim wins the match})$$

$$= 1 - \frac{98}{125}$$

$$= \frac{27}{125} \quad (1)$$

d)



$$\text{Volume} = \pi\int_0^4 x^2\,dy + \pi\int_4^{12} x^2\,dy$$

$$= \pi\int_0^4 y\,dy + \pi\int_4^{12} \frac{12-y}{2}\,dy \quad (1)$$

$$= \pi\int_0^4 y\,dy + \frac{\pi}{2}\int_4^{12} 12 - y\,dy$$

$$= \pi\left[\frac{y^2}{2}\right]_0^4 + \frac{\pi}{2}\left[12y - \frac{y^2}{2}\right]_4^{12} \quad (1)$$

$$= \pi\left[\frac{16}{2} - 0\right] + \frac{\pi}{2}\left[\left[12(12) - \frac{12^2}{2}\right] - \left[12(4) - \frac{4^2}{2}\right]\right]$$

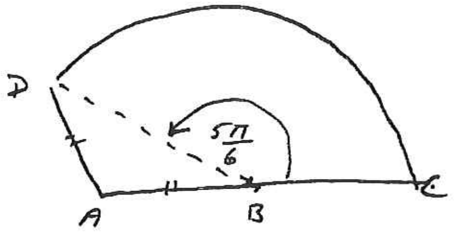
$$= 8\pi + \frac{\pi}{2}\left[(144 - 72) - (48 - 8)\right]$$

$$= 8\pi + \frac{\pi}{2}(32)$$

$= 24\pi$ units^3 (1)

Question 14.

(a)



(i) $\angle ABD = \pi - \angle CBD$ (straight angle)

$= \pi - \frac{5\pi}{6}$

$= \frac{\pi}{6}$

$\angle ADB = \pi/6$, ΔADB isosceles

$\therefore \angle DAB = \pi - \frac{2\pi}{6}$ (Angle sum of ΔADB)

$= \frac{2\pi}{3}$. (1)

(ii) Using sine rule.

$$\frac{BD}{\sin\frac{2\pi}{3}} = \frac{3}{\sin\pi/6} \quad (1)$$

$$BD = \frac{3 \times \sin\frac{2\pi}{3}}{\sin\pi/6}$$

$$= \frac{3 \times \frac{\sqrt{3}}{2}}{1/2}$$

$$= 3\sqrt{3}\text{ m.} \quad (1)$$

$$= 5.19615...$$

(iii) Area of garden ABCD

$= $ Area of ΔABD + Area sector BCD

$= \frac{1}{2} \times 3 \times 3 \times \sin 2\pi/3 + \frac{1}{2} \times (3\sqrt{3})^2 \times \frac{5\pi}{6}$

$= \frac{1}{2} \times 9 \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times 27 \times \frac{5\pi}{6}$

$= \frac{9\sqrt{3}}{4}\text{ m}^2 + \frac{135\pi}{12}\text{ m}^2$

$= 3.89711...$ (1) (1)

$\therefore$ Area of Garden $= \frac{9\sqrt{3}}{4} + \frac{45\pi}{4}$

$= 3.89711... + 35.3429...$

$= \frac{9\sqrt{3} + 45\pi}{4}\text{ m}^2$

$= 39.24\text{ m}^2$ (2dp)

(b) $y = \frac{\log_e x}{x}$, $x > 0$

$$\frac{dy}{dx} = \frac{x \cdot \frac{d}{dx}\ln x - \ln x \cdot \frac{d}{dx}x}{x^2}$$

$$= \frac{x \times \frac{1}{x} - \log_e x \times 1}{x^2}$$

$$= \frac{1 - \log_e x}{x^2} \quad (1)$$

$\frac{dy}{dx} = 0$, $\frac{1-\log_e x}{x^2} = 0$

$1 - \log_e x = 0$

$\log_e x = 1$

$x = e$

when $x = e \Rightarrow y = \frac{1}{e}$

$\therefore$ Stationary pt is $(e, \frac{1}{e})$ (1)

x	$x<e$	e	$x>e$
$\frac{dy}{dx}$	+ve	0	−ve

(1)

$\therefore$ Stationary pt $(e, \frac{1}{e})$ is a rel. max.

c) $a = 4\sin 2t$ m/s²

(i) $v = -2\cos 2t + c$ (1)

$t = 0, v = 2$

$2 = -2\cos 0 + c$

$c = 4.$ (1)

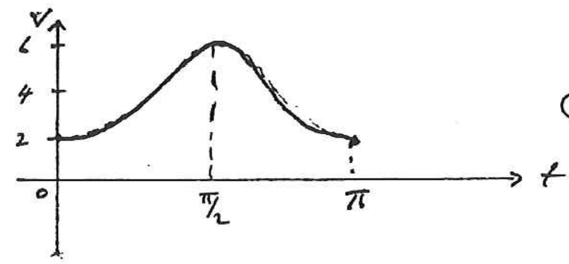
$\therefore v = 4 - 2\cos 2t$

(ii) when $v = 0$, $0 = 4 - 2\cos 2t$

$\cos 2t = 2$

No solution $\therefore$ particle never comes to rest. (1)

(iii)



(2)

(iv) distance travelled in first 4 seconds

$$= \int_0^4 4 - 2\cos 2t\,dt$$

$$= \left[4t - \sin 2t\right]_0^4 \quad (1)$$

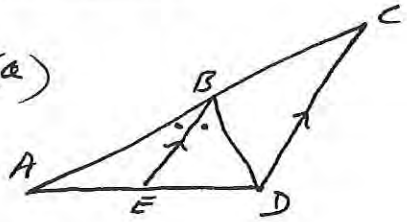
$$= (16 - \sin 8) - (0 - 0)$$

$= 15$ metres, nearest metre.

(1)

Question 15

(a)



(i) $\angle EBD = \angle BDC$
(alternate angles $BE \parallel CD$) (1)

(ii) $\angle ABE = \angle BCD$
(Corresponding angles $BE \parallel CD$) (1)
$\angle EBD = \angle ABE$ (given)
$\therefore \angle BDC = \angle BCD$
$\therefore \triangle BCD$ is isoscales (1)
(two equal angles).

(iii) $\dfrac{AE}{ED} = \dfrac{AB}{BC}$ (1)
(equal ratios on transversals by parallel lines)
$BC = BD$ (isosceles $\triangle BCD$)
$\therefore \dfrac{AE}{ED} = \dfrac{AB}{BD}$ (1)
ie $AE : ED = AB : BD$

(b) i) $P = 150 + 300e^{-0.05t}$

$\dfrac{dP}{dt} = -0.05 \times 300e^{-0.05t}$
$= -15e^{-0.05t}$ (1)

When $t = 10$

$\dfrac{dP}{dt} = -15e^{-0.5}$
$= -9.0979...$

$\therefore$ Bird population is decreasing by 9.0979... birds per year. (1)

(ii) As $t \to \infty$, $P = 150$ (1)

(iii) When $P = 200$
$200 = 150 + 300e^{-0.05t}$
$50 = 300e^{-0.05t}$
$\frac{1}{6} = e^{-0.05t}$ (1)
$\log_e \frac{1}{6} = -0.05t$
$t = 35.8351...$
During the 36th year the bird population will be eligible for inclusion. (1)

(c) i) $A_1 = P(1.005) - 4000$ (1)

(ii) $A_2 = (A_1 \times 1.005) - 4000$
$= [P(1.005) - 4000] \times 1.005 - 4000$
$= P(1.005)^2 - 4000(1 + 1.005)$ (1)
$A_3 = (A_2 \times 1.005) - 4000$
$= [P(1.005)^2 - 4000(1 + 1.005)] \times 1.005 - 4000$ (1)
$\therefore A_3 = P(1.005)^3 - 4000(1 + 1.005 + 1.005^2)$

(iii) Interest rate fixed at 9% for remainder of loan then,

$A_4 = A_3 \times 1.0075 - 4800$
$A_5 = [A_3 \times 1.0075 - 4800] \times 1.0075 - 4800$
$= A_3(1.0075)^2 - 4800(1 + 1.0075)$

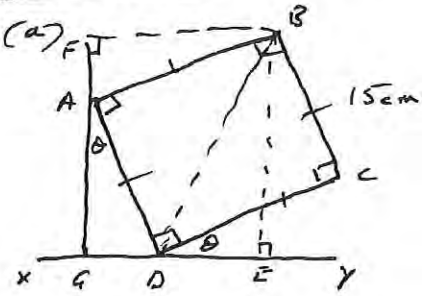
$A_6 = A_3(1.0075)^3 - 4800(1 + 1.0075 + 1.0075^2)$
$\vdots$ (1)
$A_{36} = A_3(1.0075)^{33} - 4800(1 + 1.0075 + \ldots + 1.0075^{32})$
Since $A_{36} = 0$
then $0 = A_3(1.0075)^{33} - 4800\left[\dfrac{1(1.0075^{33} - 1)}{1.0075 - 1}\right]$

$\therefore A_3 = \dfrac{640\,000(1.0075^{33} - 1)}{1.0075^{33}}$

Using part (ii), $A_3 = P(1.005)^3 - 12060.10$
$\therefore P(1.005)^3 = \dfrac{640\,000(1.0075^{33} - 1)}{1.0075^{33}} + 12060.1$
$\therefore P = \$149\,662.11$ (1)

Question 16

(a)



(i) $BD^2 = 15^2 + 15^2$

$BD^2 = 450$

$BD = 15\sqrt{2}$ (1)

(ii) $\angle BDC = \frac{\pi}{4}$ (diagonal of a square bisect). (1)

$\triangle BED$, $\sin\left(\frac{\pi}{4} + \theta\right) = \frac{BE}{15\sqrt{2}}$

$\therefore BE = 15\sqrt{2}\sin\left(\frac{\pi}{4} + \theta\right)$ (1)

(iii) In $\triangle AGD$, $\cos\theta = \frac{AG}{15}$

$\therefore AG = 15\cos\theta$ (1)

In $\triangle AFB$, $\sin\theta = \frac{AF}{15}$

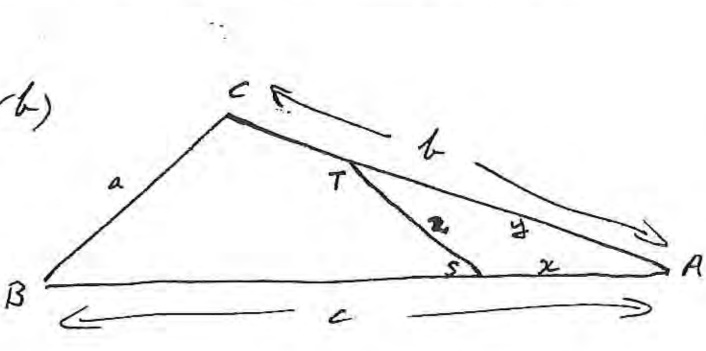
$\therefore AF = 15\sin\theta$ (1)

(iv) $BE = FG$

$15\sqrt{2}\sin\left(\frac{\pi}{4} + \theta\right) = 15\cos\theta + 15\sin\theta$ (1)

$\therefore \sqrt{2}\sin\left(\frac{\pi}{4} + \theta\right) = \cos\theta + \sin\theta$

(b)



(i) Area $\triangle ABC = \frac{1}{2} bc \sin A$

Area $\triangle AST = \frac{1}{2} xy \sin A$

Area $\triangle AST$ is half area $\triangle ABC$

$\therefore \frac{1}{2} xy \sin A = \frac{1}{2} \times \frac{1}{2} bc \sin A$

$\therefore xy = \frac{1}{2} bc$ (1)

(ii) Using cosine rule in $\triangle AST$,

$z^2 = x^2 + y^2 - 2xy\cos A$ (1)

since $xy = \frac{1}{2} bc \Rightarrow y = \frac{bc}{2x}$

$\therefore z^2 = x^2 + \left(\frac{bc}{2x}\right)^2 - 2x\left(\frac{bc}{2x}\right)\cos A$ (1)

$z^2 = x^2 + \frac{b^2c^2}{4x^2} - bc\cos A$

(iii) $z^2 = x^2 + \frac{b^2c^2}{4} x^{-2} - bc\cos A$

$\frac{d(z^2)}{dx} = 2x - \frac{2b^2c^2}{4} x^{-3}$ (1)

$= 2x - \frac{b^2c^2}{2x^3}$

Stat. pts when $\frac{d}{dx}(z^2) = 0$

ie $2x - \frac{b^2c^2}{2x^3} = 0$

$4x^4 - b^2c^2 = 0$

$4x^4 = b^2c^2$

$x^4 = \frac{b^2c^2}{4}$

$x^2 = \frac{bc}{2}$ $(x^2 \geq 0)$ (1)

$x = \sqrt{\frac{bc}{2}}$ $(x > 0)$

$\frac{d(z^2)}{dx} = 2x - \left(\frac{b^2c^2}{2}\right)x^{-3}$

$\therefore \frac{d^2(z^2)}{dx^2} = 2 + \frac{3b^2c^2}{2} x^{-4}$ (1)

$= 2 + \frac{3b^2c^2}{2x^4}$

> 0 for all x

$\therefore$ Minimum value of z^2 when $x = \sqrt{\frac{bc}{2}}$

(iv) $z^2 = x^2 + \frac{b^2c^2}{4x^2} - bc\cos A$

and $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\therefore z^2 = x^2 + \frac{b^2c^2}{4x^2} - bc\left(\frac{b^2+c^2-a^2}{2bc}\right)$

when $x = \sqrt{\frac{bc}{2}}$,

PTO.

$$z^2 = \frac{bc}{2} + \frac{b^2c^2}{4\left(\frac{bc}{2}\right)} - \left(\frac{b^2+c^2-a^2}{2}\right) \quad (1)$$

$$= \frac{bc}{2} + \frac{bc}{2} - \left(\frac{b^2+c^2-a^2}{2}\right)$$

$$= bc - \left(\frac{b^2+c^2-a^2}{2}\right)$$

$$= \frac{a^2 - b^2 + 2bc - c^2}{2}$$

$$= \frac{a^2 - (b^2 - 2bc + c^2)}{2}$$

$$= \frac{1}{2}\left[a^2 - (b-c)^2\right] \quad (1)$$

$$= \frac{1}{2}\left[a - (b-c)\right]\left[a + (b-c)\right]$$

$$= \frac{1}{2}\left[a - b + c\right]\left[a + b - c\right]$$

$$= \frac{1}{2}(a+b+c-2b)(a+b+c-2c)$$

$$= \frac{1}{2}(P-2b)(P-2c) \quad \text{where } P = a+b+c$$

$$z^2 = \frac{(P-2b)(P-2c)}{2}$$

$$\therefore \quad z = \sqrt{\frac{(P-2b)(P-2c)}{2}} \quad (1)$$

MULTIPLE CHOICE SOLUTIONS.

QUESTION 1.

$\frac{e^4}{7} = \frac{54.59815003}{7}$

$= 7.79973\ldots$

$= 7.80$ 3SF (D)

QUESTION 2. $(1-2x)(3+x)=0$

$\therefore 1-2x=0$ or $3+x=0$

$\therefore x=\frac{1}{2}$ or $x=-3$ (B)

QUESTION 3.

$y=e^{2x}$, $y=4x-1$

$y'=2e^{2x}$

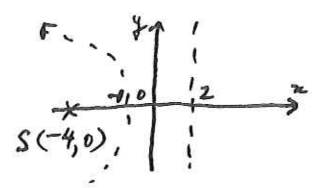
$\therefore 2e^{2x}=4$

$e^{2x}=2$

$2x=\ln 2$

$x=\frac{1}{2}\ln 2$ (A)

QUESTION 4.



parabola, $(y-k)^2=-4a(x-h)$

Vertex $(-1,0)$ and $a=3$. $\therefore (y-0)^2=-4(3)(x+1)$

ie $y^2=-12(x+1)$ (B)

QUESTION 5.

$x=t^2-3t$

$\therefore v=2t-3$

when $t=1 \Rightarrow v=-1$ (A)

acceleration, $a=2$

$\therefore$ particle moving to left (v, negative) with decreasing speed. (a, positive)

Question 6.

region above $y=e^{-x}$, $y \geq e^{-x}$

region below $y=2-x$, $y \leq 2-x$

$\therefore$ Shaded region described by

$y \geq e^{-x}$ and $y \leq 2-x$ (B)

Question 7.

$\sqrt{5}+3\sqrt{5}+5\sqrt{5}+\ldots=225\sqrt{5}$

Series is AP with $d=2\sqrt{5}$ and $a=\sqrt{5}$

$\therefore 225\sqrt{5}=\frac{n}{2}\left[2\sqrt{5}+(n-1)2\sqrt{5}\right]$

$450\sqrt{5}=n\left[2n\sqrt{5}\right]$

$\therefore n^2=225$

$n=15$ (A)

(Note: $\sqrt{5}$ could be ignored in series.)

Question 8.

From the graph of $y=f'(x)$,

to the left of D tangent +ve,

to the right of D tangent +ve.

at D tangent horizontal.

$\therefore$ D represents a horizontal pt of inflexion on $y=f(x)$.

(D)

Question 9.

$f(x)=\frac{1}{3x}=\frac{1}{3}x^{-1}$

$f'(x)=-\frac{1}{3}x^{-2}$

$f'(2)=-\frac{1}{3}\times\frac{1}{4}$

$=-\frac{1}{12}$ (A)

Question 10.

$y=\sin x$ and $y=\cos x$

intersect when $\sin x=\cos x$

ie $\tan x=1$

$x=\pi/4,\ 5\pi/4$

$\therefore$ Area shaded is represented by $\int_{\pi/4}^{5\pi/4}(\sin x-\cos x)\,dx$.

(C)